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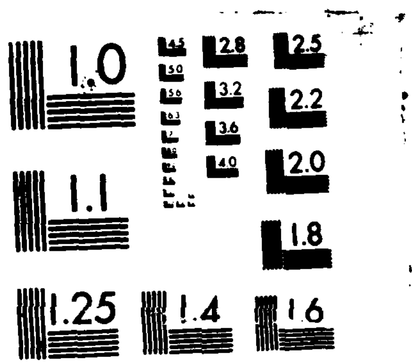

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A computational study of radiation heat transfer in a combustor having fuel spray combustion is reported in the present paper. When the distribution of radiatively participating species is known in either a normal distribution or an exponential distribution in any direction of cylindrical coordinates with respect to a referenced location, e.g., in a form, $F = f\exp\cancel{(-a\rho-b\phi^2-cz^2)}$, where f, a, b, and c are distribution constants for individual species, the governing equation of radiation heat transfer without scattering in such a over

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system can be solved with relative ease by using the herein reported method. Several results from the present authors' work are explained that are employed for implementing the solution: a new coordinate transformation method to find the species distribution along individual optical paths centered at any chosen location in the combustor; a new formula for the adiabatic flame temperature in a logarithmic function expressed in terms of relevant variables; a new integral function facilitating solutions for various radiation equations; a new inverse error function enabling some exact solution of the governing equation of radiation heat transfer, etc.

An exact solution is found for both optically thin and thick media. And a numerical solution is offered for optically medially thin medium. The present analytical method is applied to radiation heat transfer in a direct injection-type diesel engine combustion chamber. Some representative results are discussed that have been obtained from a parametric analysis of its radiation heat transfer.

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An Analytical Method of Non-Gray Three-Dimensional Radiation Heat Transfer in Spray Combustion

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ABSTRACT

A computational study of radiation heat transfer in a combustor having fuel spray combustion is reported in the present paper. When the distribution of radiatively participating species is known in either a normal distribution or an exponential distribution in any direction of cylindrical coordinates with respect to a referenced location, e.g., in a form, $F = f \exp(-ar-b\phi^2-cz^2)$, where f, a, b, and c are distribution constants for individual species, the governing equation of radiation heat transfer without scattering in such a system can be solved with relative ease by using the herein reported method. Several results from the present authors' work are explained that are employed for implementing the solution: a new coordinate transformation method to find the species distribution along individual optical paths centered at any chosen location in the combustor; a new formula for the adiabatic flame temperature in a logarithmic function expressed in terms of relevant variables; a new integral function facilitating solutions for various radiation equations; a new inverse error function enabling some exact solution of the governing equation of radiation heat transfer, etc.

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Nomenclature

- a — Species distribution constant or $(kT/hc)\int_0^r \kappa_0 \, dr$
- b — Species distribution constant
- c — Species distribution constant or speed of light
- E — Plank radiation function
- f — Species distribution constant
- h — Plank's constant
- I — Radiation Intensity
- k — Boltzman constant
- q — Radiation heat flux
- r — Optical path
- R — *Radius of cylinder*
- T — Temperature
- ρ, ϕ, z — Components in cylindrical coordinate
- r, θ, ζ — Components in spherical coordinate
- κ — Volume absorptance
- λ — Wavelength
- τ — Optical depth
- σ — Stefan-Boltzman constant

Subscripts

- a — Adiabatic
- b — Blackbody
- d — Detector
- i — ith subrange in r-domain
- j — jth subrange in λ-domain
- λ — Spectral
- O — Foot of the optical path

INTRODUCTION

When an equation of radiative heat transfer is solved for predicting the heat flux incident upon various locations over the surface of a combustor having flame plumes, several difficult problems are encountered. In order to briefly review the difficulties, a combustor having a jet flame is considered as shown in Fig. 1. The heat flux through an optical path (r,θ,ζ) centered at location D may be calculated by solving the governing equation of radiation neglecting scattering effect,

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$$q_{\theta\zeta} = \int_0^\infty I_\lambda(0) \cos\theta \, d\lambda \qquad (1)$$

when the spectral radiation intensity, $I_\lambda(r)$ is represented as

$$I_\lambda(r) = \int_r^{r_0} \frac{\kappa_\lambda(r')}{\pi} e_{b\lambda} \exp(-\int_r^{r'} \kappa_\lambda(r'')dr'')dr' \qquad (2)$$

where κ_λ is the spectral volume absorptance at (r,θ,ζ) and $e_{b\lambda}$ is the spectral blackbody radiation. The total radiation flux on location D, then, is calculated by integrating $q_{\theta\zeta}$ over the entire hemispherical volume faced by the location. Among the main difficulties in implementing the above computation are that κ_λ can be determined only when the distributions of radiatively participating species and temperature are known along the individual optical paths and that I_λ is spectrum-dependent.

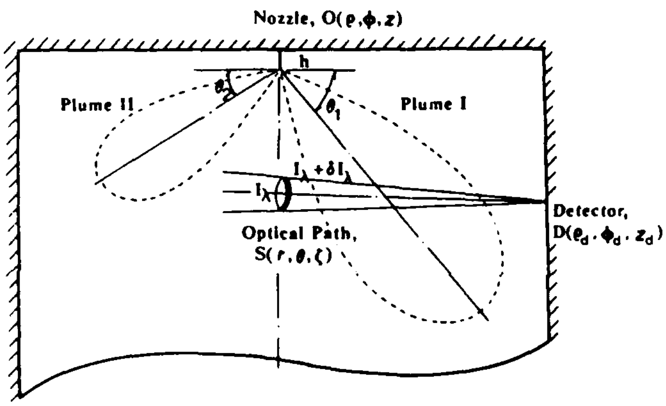

Fig. 1 A Combustor with Jet Flames.

In order to obtain a better solution of Eq. (1), some new methods were proposed for overcoming the abovementioned difficulties when the authors computed radiation heat transfer in a direct injection-type diesel engine: a coordinate transformation technique to find the species distribution along individual optical paths when the distribution is given with respect to a chosen location, e.g., the injection nozzle hole [1]; a computation method of finding the spectral volume absorptances of soot and gas for their mixtures [1,2] by using reported results [3,4,5] and new analytical and numerical methods of solving Eq. (1), which are considered in the present paper. This paper, therefore, may partially serve as a summary report of the authors' modeling study of radiation heat transfer in a combustor having flame jets, in addition to reporting new analytical solutions of the radiation heat transfer equation.

Elaborating on the species distribution in the combustion chamber, it was found to be conveniently described by an equation in cylindrical coordinates with respect to the injection nozzle [1] when the distribution is known in either a normal (or a skewed normal) or an exponential form in all directions of the cylindrical coordinate system, e.g., in a form,

$$F = f \exp(-a\rho - b\phi^2 - cz^2) \qquad (3)$$

where F represents either fuel/air ratio or soot concentration or CO_2 or H_2O concentration; f, a, b and c are constants to be separately determined for each medium by either experimental or theoretical means. The expression is considered as to reasonably well describe the species distribution of non-axisymmetric plumes in view of some reported results from both theoretical and experimental studies [6,7]. The new expression offers various advantages, e.g., the expression can be used to describe the spray plume in swirl motion by including its effect in the ϕ-direction and it facilitates the sought solution of Eq. (1) [1]. When the species distribution with respect to a referenced position in the combustor is known, the species distribution along the optical path, r, of any chosen direction (r,θ,ζ) and location D (ρ_d,ϕ_d,z_d) may be found as [1],

$$F_o = f_d \exp(-[(r-r_o)/r_w]^2 \qquad (4)$$

where f_d, r_o and r_w are functionally related to θ, ζ, ρ_d, ϕ_d, z_d, f, a, b, and c. This distribution equation, then, may be used for determining the volume absorptance according to the reported methods [3,4,5] as

$$\kappa_o = \kappa_1 \exp(-c_1(r-r_1)^2) \qquad (5)$$

The temperature distribution, T, needed in the above equation and $e_{b\lambda}$ for Eq. (1) may be determined by using the distribution of burned fuel/air ratio, ω, from Eq. (3). This can be achieved by using our new formula for the adiabatic flame temperature [8],

$$T = T_o + T_1 \ln\omega + T_2 (\ln\omega)^2 \qquad (6)$$

where T_o, T_1 and T_2 are functionally related to the reaction pressure, the initial mixture temperature and the number of carbon atoms of the fuel molecule. Even with the above rearrangement and simplification of terms in Eq. (1), its solution was not readily found until some new mathematical techniques were employed as explained next.

ANALYTICAL AND NUMERICAL SOLUTIONS

The solution of Eq. (1) may be considered for the following three cases: (1) the optically thin medium; (2) optically thick medium and (3) optically medially thick medium. The solution for each case will be explained upon further simplifying the governing equation as follows. Introducing new terms,

$$\kappa_o = \kappa_\lambda \cdot \lambda \ [3,9], \quad x = hc/k\lambda T \quad \text{and} \quad a = \frac{kT}{hc}\int_0^r \kappa_o \, dr,$$

where h is Planck's constant, c the speed of light and k the Boltzmann constant, Eq. (1) can be rewritten for directional radiation intensity as,

$$I_{\theta\zeta} = q_{\theta\zeta}/\cos\theta,$$

$$I_{\theta\zeta} = \int_0^{r_0} \kappa_o \int_0^\infty \frac{1}{\lambda} e_{b\lambda} e^{-\frac{1}{\lambda}\int_0^r \kappa_o dr} d\lambda \, dr$$

$$= \frac{15}{\pi^5}\frac{\sigma k}{hc} \int_0^{r_0} \kappa_o T^5 \int_0^\infty \frac{x^4 e^{-ax}}{e^x - 1} dx \, dr.$$

During the process of simplifying the above equation, the authors discovered a new useful integration function [10],

$$\int_{x_1}^{x_2} \frac{e^{-ax}x^m}{e^x-1}\,dx = F(m,a,x_1) - F(m,a,x_2), \qquad (7)$$

$$F(m,a,x) =$$

$$\sum_{n=1}^{\infty} \{e^{-(n+a)x} \sum_{i=0}^{m} [\frac{\Gamma(m+1)}{\Gamma(m-i+1)} (n+a)^{-i-1} x^{m-i}]\},$$

where $\Gamma(m+1) = m!$

Note that Eq. (7) can also be used for determining a solution in closed form of the black body radiation function, $F_{0-\lambda T}$ [11],

$$F_{0-\lambda T} = \int_0^{\lambda T} \frac{E_{b\lambda}(T)}{\sigma T^4}\, d(\lambda T)$$

$$= \frac{15}{\pi^4} \sum_{n=1}^{\infty} [\frac{e^{-nx}}{n} (x^3 + \frac{3x^2}{n} + \frac{6x}{n^2} + \frac{6}{n^3})] .$$

From Eq. (7), since one can find

$$\int_0^{\infty} \frac{x^4 e^{-ax}}{e^x-1}\,dx = \sum_{n=1}^{\infty} \frac{4!}{(n+a)^5},$$

the governing equation is simplified as

$$I_{\theta\zeta} = \frac{360\ \sigma\kappa}{\pi^5\ hc} \sum_{n=1}^{\infty} \frac{1}{n^5} \int_0^{r_0} \frac{\kappa_0\ T^5}{(1+a/n)^5}\, dr .$$

With the definition of the optical depth, $\tau = \int_0^r \kappa_0\, dr$, the equation may be rewritten in a tidier form,

$$I_{\theta\zeta} = \frac{360\ \sigma\kappa}{\pi^5\ hc} \sum_{n=1}^{\infty} \frac{1}{n^5} \int_0^{\tau_0} \frac{T^5}{(1+\frac{kT\tau}{nhc})^5}\, d\tau \qquad (8)$$

Note that κ_0 in the equation is assumed constant with respect to λ, which appears valid when soot generated radiation dominates [19]. The solution of this equation is considered for the previously mentioned three cases as follows.

<u>OPTICALLY THIN MEDIUM:</u> When τ approaches zero, i.e., in the optically thin limit, an exact solution for Eq. (8) was found. In this case, Eq. (8) becomes

$$I_{\theta\zeta} = \frac{360\ \sigma\kappa}{\pi^5 hc} \sum_{n=1}^{\infty} \frac{1}{n^5} \int_0^{\tau_0} T^5 (1 - \frac{5kT\tau}{nhc})\, d\tau \qquad (9)$$

For finding the solution of the above equation, since T is functionally related to the optical depth, τ, and the burned fuel/air ratio, ω, and since ω may be found as $\omega = \omega_1 e^{-\bar{r}^2}$ from Eq. (4), Eq. (6) may be written as

$$T = T_0 + T_1 (\ln \omega_1 - \bar{r}^2) + T_2 (\ln \omega_1 - \bar{r}^2)^2$$

$$= T_3 - T_4 \bar{r}^2 + T_2 \bar{r}^4 \qquad (10)$$

where, $T_3 = T_0 + T_1 \ln \omega_1 + T_2(\ln \omega_1)^2$ and

$T_4 = T_1 + 2T_2 \ln(\omega_1)$.

In addition, for obtaining the solution of Eq. (8), it was necessary to find the relationship between τ and r. From definition and Eq. (5), the optical depth, τ, is rewritten as

$$\tau = \kappa_1 \int_0^r e^{-c_1(r-r_1)^2}\, dr$$

$$= \frac{\kappa_1}{2} \sqrt{\frac{\pi}{c_1}} \{\mathrm{erf}\ [\sqrt{c_1}(r-r_1)] + \mathrm{erf}\ (\sqrt{c_1}\ r_1)\}$$

$$\text{or, } \mathrm{erf}\ [\sqrt{c_1}(r-r_1)] = \frac{2\pi}{\kappa_1} \sqrt{\frac{c_1}{\pi}}\ \tau - \mathrm{erf}\ (\sqrt{c_1}\ r_1). \qquad (11)$$

In view of Eq. (10) and for further simplifying Eq. (11), a variable transformation was employed to obtain

$$\bar{r} = \sqrt{c_1}\ (r-r_1),$$

$$\bar{\tau} = \frac{2}{\kappa_1} \sqrt{\frac{c_1}{\pi}} [\tau - \frac{\kappa_1}{2} \sqrt{\frac{\pi}{c_1}}\ \mathrm{erf}\ (\sqrt{c_1}\ r_1)]$$

$$= d_1\ [\tau - \tau_1] . \qquad (12)$$

Then Eq. (11) may be written as

$$\mathrm{erf}\ (\bar{r}) = \bar{\tau}$$

$$\text{or,} \quad \bar{r} = \mathrm{erf}^{-1}(\bar{\tau}) \qquad (13)$$

where, erf^{-1} represents the inverse error function, whose exact solution is found as follows (the details of its derivation are explained in the Appendix):

$$\bar{r} = \mathrm{erf}^{-1}(\bar{\tau})$$

$$= \frac{\sqrt{\pi}}{2} (\bar{\tau} + \frac{\pi}{2.3!} \bar{\tau}^3 + \frac{7\pi^2}{4.5!} \bar{\tau}^5 + \ldots.) \qquad (14)$$

Next, the above equation is introduced into Eq. (10) to find

$$T = T_3 - \frac{T_4 \pi}{4} (\bar{\tau} + \frac{\pi}{2.3!} \bar{\tau}^3 + \frac{7\pi^2}{4.5!} \bar{\tau}^5 + \ldots.)^2$$

$$+ \frac{T_2 \pi^2}{16} (\bar{\tau} + \frac{\pi}{2.3!} \bar{\tau}^3 + \frac{7\pi^2}{4.5} \bar{\tau}^5 + \ldots.)^4$$

$$= T_3 - \frac{\pi}{4} T_4\ \bar{\tau}^2 + (-\frac{\pi^2 T_4}{4.3!} + \frac{\pi^2}{16} T_2)\ \bar{\tau}^4$$

$$= T_3 - T_5\ \bar{\tau}^2 + T_6\ \bar{\tau}^4 \qquad (15)$$

where, $T_5 = \frac{\pi}{4} T_4$ and $T_6 = (-\frac{\pi^2 T_4}{4.3!} + \frac{\pi^2}{16} T_2)$.

From Eq. (15), one may also find the following in order to use them in Eq. (9):

$$T^5 = T_3^{\,5} - 5T_3^{\,4}\,T_5\,\bar{\tau}^2 + (10T_3^{\,3}\,T_5^{\,2} + 5T_3^{\,4}\,T_6)\,\bar{\tau}^{-4},$$

$$T^6 = T_3^{\,6} - 6T_3^{\,5}\,T_5\,\bar{\tau}^2 + (15T_3^{\,4}\,T_5^{\,2} + 6T_3^{\,5}\,T_6)\,\bar{\tau}^{-4}.$$

Finally, the above equations and Eq. (12) are introduced into Eq. (9), with a change of variables $\bar{\tau} = d_1(\tau-\tau_1)$ and $d\tau = 1/d_1\, d\bar{\tau}$, to obtain

$$I_{\theta\zeta} = \frac{360\ \sigma k}{\pi^5\ hc} \sum_{n=1}^{\infty} \frac{1}{n^5} \int_{-d_1\tau_1}^{d_1(\tau_0-\tau_1)} \frac{1}{d_1} \Big[\Big(T_3^{\,5} - \frac{5k\tau_1 T_3^{\,6}}{nhc}\Big) - \frac{5kTT_3^{\,6}}{nhcd_1}\,\bar{\tau}$$

$$+ \Big(\frac{30kT_1T_3^{\,5}T_5}{nhc} - 5\,T_3^{\,4}\,T_5\Big)\,\bar{\tau}^2 + \frac{30kTT_3^{\,5}T_5}{nhcd_1}\,\bar{\tau}^3$$

$$+ \Big(10T_3^{\,3}T_5^{\,2} + 5\,T_3^{\,4}\,T_6$$

$$- \frac{75k\tau_1T_3^{\,4}T_5^{\,2}}{nhc} + \frac{30k\tau_1T_3^{\,5}T_6}{nhc}\Big)\,\bar{\tau}^{4}$$

$$- \frac{5kT}{nhcd_1}\,(15\,T_3T_5^{\,2} + 6\,T_3^{\,5}\,T_6)\bar{\tau}^5\Big]\,d\bar{\tau} \qquad (16)$$

Consequently, Eq. (16) can be used for finding an exact solution of the equation of radiation for an optically thin medium in a combustor with flame plumes as,

$$I_{\theta\zeta} = \frac{360\ \sigma k}{\pi^5\ hc} \sum_{n=1}^{\infty} \frac{1}{n^5 d_1} \Big[\Big(T_3^{\,5} - \frac{5k\tau_1T_3^{\,6}}{nhc}\Big)\, d_1\tau_0$$

$$- \frac{5kTT_3^{\,6}}{2nhcd_1}\,\big(d_1^{\,2}(\tau_0 - \tau_1)^2 - d_1^{\,2}\tau_1^{\,2}\big)$$

$$+ \Big(\frac{10kT_1T_3^{\,5}T_5}{nhc} - \frac{5}{3}\,T_3^{\,4}T_5\Big)\big(d_1^{\,3}(\tau_0 - \tau_1)^3 - d_1^{\,3}\,\tau_1^{\,3}\big)$$

$$+ \frac{15kTT_3^{\,5}T_5}{2nhcd_1}\,\big(d_1^{\,4}\,(\tau_0 - \tau_1)^4 - d_1^{\,4}\tau_1^{\,4}\big)$$

$$+ \Big(2\,T_3^{\,3}T_5^{\,2} + T_3^{\,4}T_6 - \frac{15k\tau_1T_3^{\,4}T_5^{\,2}}{nhc} + \frac{6k\tau_1T_3^{\,5}T_6}{nhc}\Big)$$

$$\big(d_1^{\,5}(\tau_0 - \tau_1)^5 - d_1^{\,5}\tau_1^{\,5}\big)$$

$$- \frac{5kT}{nhcd_1}\,\Big(\frac{5}{2}\,T_3\,T_5^{\,2} + T_3^{\,5}\,T_6\Big)$$

$$\big(d_1^{\,6}(\tau_0 - \tau_1)^6 - d_1^{\,6}\tau_1^{\,6}\big)\Big]. \qquad (17)$$

Optically thick medium: An exact solution can also be found in an optically thick medium: assuming the volume absorptance κ is 0 for $r < r_0$, and κ_0 for $r_0 < r < r_0+\varepsilon$, one can rewrite Eq. (2) as

$$I_{\theta\zeta} = \int_0^{r_0+\varepsilon} \frac{\kappa}{\pi}\, e_b \exp\Big(-\int_0^{r'} \kappa\, dr''\Big)dr'$$

This equation is further solved to find the result,

$$I_{\theta\zeta} = \int_{r_0}^{r_0+\varepsilon} \frac{\kappa_0}{\pi}\, e_b \exp\Big(-\int_{r_0}^{r'} \kappa_0 dr''\Big)dr'$$

$$= \frac{\kappa_0 e_b}{\pi} \int_{r_0}^{r_0+\varepsilon} \exp(-\kappa_0(r'-r_0))dr'$$

$$= \frac{\sigma T^4}{\pi}\,[1-e^{-\tau}] \qquad (18)$$

Optically medially thin medium: When the system holds an optically medially thin (or thick) medium, Eq. (2) cannot analytically be solved as the above discussed cases; rather, a numerical method was employed for it. Without going into the details of its derivation, but by knowing that the volume absorptance $\kappa(r)$ can be found from Eq. (5), the following result was used for the solution.

$$I_{\theta\zeta} = \frac{15\ \sigma k}{\pi^5\ hc} \int_0^{r_0}\Big(\int_0^{\infty}\kappa_0\, T^5\, \Big(\frac{x^4 e^{-ax}}{e^x - 1}\, dx\Big)\, dr$$

$$= \frac{15\ \sigma k}{\pi^5\ hc} \sum_{i=1}^{M} \int_{\tau_{i-1}}^{\tau_i} \sum_{j=1}^{J} \kappa_{0ij}\, T^5[F(4,a,x_{j-1})$$

$$- F(4,a,x_j)]\, dr$$

where, κ_{0ij} is the volume absorptance defined in the ith subrange of r-domain and the jth subrange of λ-domain. Then,

$$I_{\theta\zeta} = \frac{15\ \sigma k}{\pi^5\ hc} \sum_{i=1}^{M} \int_{\tau_{i-1}}^{\tau_i} \sum_{j=1}^{J} T^5[F(4,a,x_{j-1})$$

$$- F(4,a,x_j)]\, d\tau_{ij} \qquad (19)$$

where, $d\tau_{ij} = \kappa_{0ij}\, dr$. The numerical results of Eq. (19) were obtained using Caussian integration method.

RESULTS AND CONCLUSION

The present computational method enables an extensive parametric study of radiation heat transfer in a combustor with flame jets [2]. Since this paper is to primarily report the analytical methods of computation by using the authors' several new findings reported elsewhere [1, 2, 8, 10, 11], only some representative results obtained for a direct injection-type diesel engine are presented here.

Note that since our computer program to numerically implement Eq. (1) was developed prior to finding the exact solutions that are reported here and since some comparison between the results obtained using the program and those using the exact solutions was satisfactory, the results discussed next were mainly obtained using our available computer program. In the computation, in order to be consistent and to have better mutual comparisons, the same computational conditions employed for the previous results are considered here. The numerical values of f in Eq. (3) were for soot, H_2O, CO_2 and fuel/air ratio 8×10^{-6}, 0.01, 0.01 and 1.0, respectively; the distribution constants, a, b, and c were, for these species, 0.6, 2.36 and 2.0, accordingly. Note again that these constants and the distribution made in each coordinate of Eq. (3) can be determined by computational or experimental methods, or arbitrarily chosen for parametric studies. The engine details for the computation were: the chamber surface temperature, 500°K; the number of spray plumes, 4; the piston radius, R = 4.92 cm; the combustion chamber bowl radius, 3.44 cm; the compression ratio, 23; the engine speed, 1000 rpm; the surface spectral emissivity, 0.95; the fuel composition, $C_{16}H_{34}$; the time of computation in a cycle, 40 degrees after top dead center. The results are shown in normalized forms: normalized radiation heat transfer, $Q/\sigma T^4$, where T is 2400°K and σ is the Stefan-Boltzmann constant; normalized directional intensity, $I_{\theta\zeta}/\pi\sigma T^4$; normalized spectral radiation heat transfer, $Q_\lambda/\sigma T^4$.

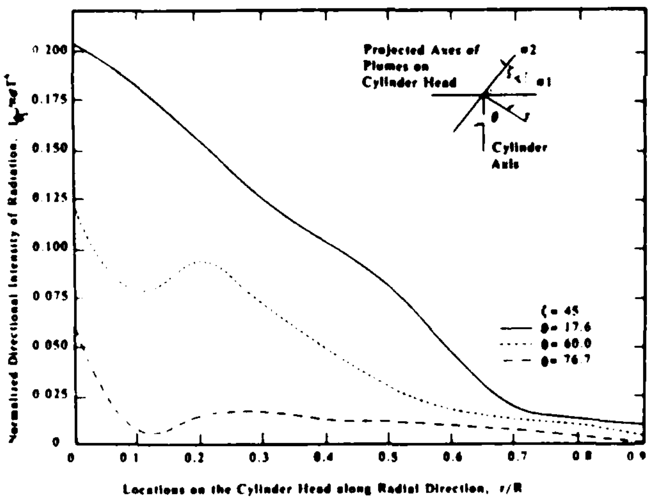

Fig. 2 Directional Radiation Heat Flux Incident on Cylinder Head Surface in Radial Direction.

The radiation heat transfer through a solid angle incident on the cylinder head, are shown in Fig. 2. When there are four flame jets in the combustion chamber, radiation heat transfer flux incident on the locations along the radial direction varies depending upon the direction in the hemispherical volume of integration. The schematic drawing in the figure shows the spherical coordinates identifying the direction of solid angles. Note that, for cases $\theta = 60°$ and $76.7°$, heat transfer on r/R < 0.1 is largely from plume #1 and that beyond r/R = 0.1 from both plumes #1 and #2. The directional radiation intensity, $I_{\theta\zeta}$ was combined with the spectral emissivity of the chamber wall, $\varepsilon_\lambda = 0.95$,

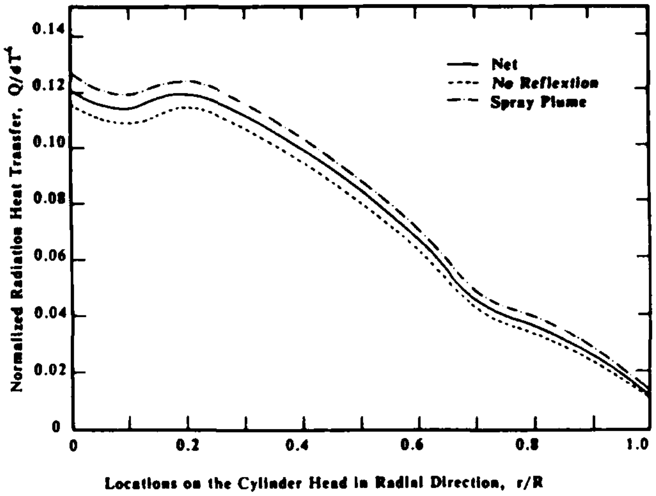

Fig. 3 Radiation Heat Transfer through Cylindrical Head.

to obtain radiation heat transfer through the cylinder head by integrating as, $Q = \int I_{\theta\zeta} \cos\theta \sin\theta \, d\theta \, d\zeta$ (Fig. 3). The net heat transfer computed with inclusion of wall emission at 500K, reflection from the opposite-side walls are represented by "Net"; those obtained excluding the reflection are denoted by "No reflection"; and the heat flux incident on the surface without including the wall effect are shown by "Spray plume". Explaining the results, the rapid reduction in heat transfer around r/R = 0.7 is caused by the presence of the piston bowl. The double hump in the plot may be surprising. In order to clarify this, an additional computation made for solid angle of zenith angle, $\theta = 39.7°$, at various azimuthal angles and locations on the cylinder head, is shown in Fig. 4. From the results, one finds that while the radiation heat flux incident on locations around the nozzle (r/R = 0) is from all of the four plumes, the flux on those near r/R = 0.1 is from one plume, i.e., the nearest

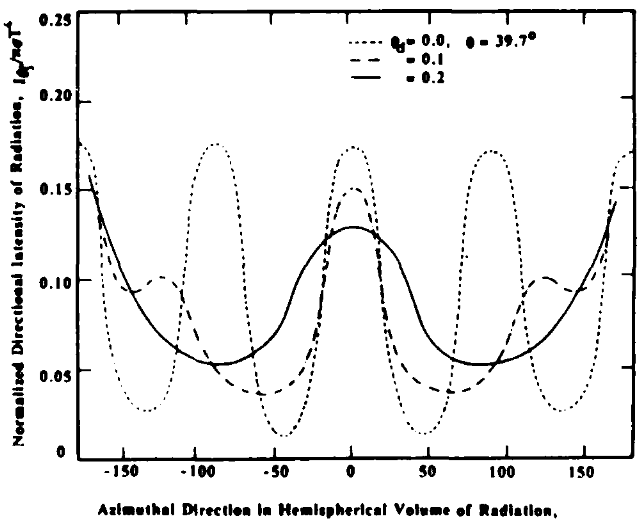

Fig. 4 Directional Intensity of Radiation Incident on Cylinder Head through Various Directions of Solid Angle.

plume to the location. If the computation is made for a single plume, the heat transfer should have asymptotically been approaching zero at $r/R = 0$. This trend is qualitatively compared with the experimental results obtained by using a single plume [12].

Since the distribution constants being used for Eq. (3) in the present analysis were chosen based on an in-cylinder soot measurement [13], it seems of interest to compute radiative heat transfer contributed by in-cylinder soot and gaseous species, i.e., mainly CO_2 and H_2O. Fig. 5 shows their spectrum-resolved radiation

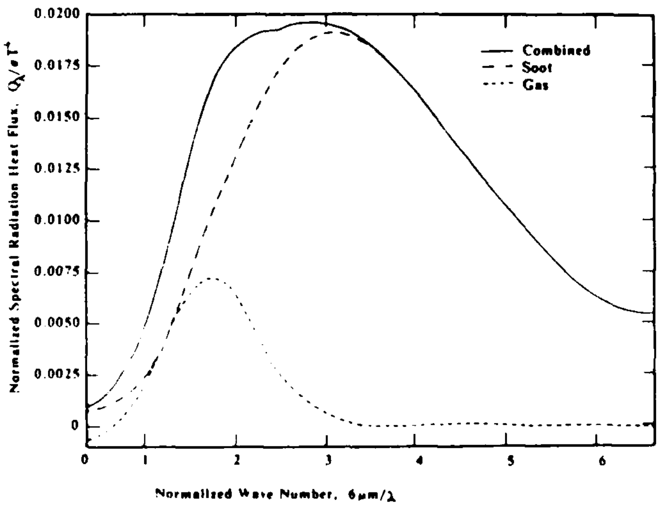

Fig. 5 Radiation Heat Transfer Contributed by Soot and Gaseous Species and their Combination.

heat transfer smoothed over individual wave bands in normalized form, $Q_\lambda/\sigma T^4$, with respect to normalized wave number, 6 μm/λ. It appears clear that thermal radiation transmission caused by the presence of soot almost dominates the entire radiation process and the radiation by gaseous species is very small. The great portion of radiation by gas found around 6μm/λ = 1.8 is explained by the strong emission bands of H_2O at λ = 2.7μm and CO_2 at λ = 2.7 and 4.3 μm. Since the spectral absorptance of a surface can be incorporated with results like those in Fig. 5, the analysis may be made for combustors having various surface coatings with known spectral emissivity.

The main issue associated with the use of the present analytical method of radiation heat transfer may be justification of the species distribution described by Eq. (3). Since the distribution widely varies depending upon the fuel injector and combustor condition, it is difficult to exactly describe the variations by a single equation. Reported results suggest, however, the individual species distribution may be simplified by using an exponential or normal (or skewed normal) distribution in each coordinate component of a cylindrical system, e.g., Eq. (3). It is found in the present study that the radiation heat analysis is greatly simplified when the distribution is made in such a form without resorting to the rather complex zonal method or the use of the geometric factors. Further, the use of the present method readily enables a parametric analysis of radiation heat transfer, e.g., the selection of an injector suitable for a chosen combustor system.

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Appendix:

AN INVERSE ERROR FUNCTION

The error function, erf(x), although defined in several ways, is essentially equivalent to the following expression:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{n!(2n+1)} \quad \text{(A-1)}$$

The equation is the integral of the so-called Gaussian or normal function and occurs frequently in the study of the general theory of probability [14], the analysis of transient heat flow in a semi-infinite solid [15], and the computation of radiation heat transfer with radiatively participating media [10], etc. For meeting the needs of such studies, the error function, its derivatives and integrals have been tabulated [16]. As found in the text, it was needed to obtain the inverse form of the error function, i.e., in Eq. (A-1), the value of x for a corresponding erf(x). The following derivation shows a new inverse error function in closed form:

When the error function, erf(x), is set equal to y, i.e., $y = \operatorname{erf}(x)$, its inverse form may be expressed, for the convenience of discussion,

$$x = \operatorname{erf}^{-1}(y) \quad \text{(A-2)}$$

or,

$$x = \operatorname{fre}(y). \quad \text{(A-3)}$$

By Taylor's expansion method, the inverse error function can be expressed as

$$x = \sum_{n=0}^{\infty} \frac{x^{(n)}(0)}{n!} y^n \quad \text{(A-4)}$$

where, $x^{(n)}(0)$ is the nth derivative of x with respect to y at $y = 0$. In order to find $x^{(n)}(0)$, the following steps are taken.

$$x'(y) = \frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$$

$$= \left\{\frac{d}{dx}\left[\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt\right]\right\}^{-1}$$

$$= \frac{\sqrt{\pi}}{2} e^{x^2},$$

$$x''(y) = \frac{d}{dy}[x'(y)] = \frac{d}{dx}[x'(y)]\left[\frac{dy}{dx}\right]^{-1}$$

$$= \frac{\pi}{2} x\, e^{2x^2},$$

$$x^{(3)}(y) = \frac{d}{dy}[x''(y)] = \frac{d}{dx}[x''(y)]\left[\frac{dy}{dx}\right]^{-1}$$

$$= \left[\frac{\pi}{2}(1 + 4x^2)\, e^{2x^2}\right]\left[\frac{\sqrt{\pi}}{2} e^{x^2}\right]$$

$$= \frac{\pi^{3/2}}{4}(1 + 4x^2)\, e^{3x^2},$$

$$x^{(4)}(y) = \frac{\pi^2}{4}(7x + 12x^3)\, e^{4x^2},$$

$$x^{(5)}(y) = \frac{\pi^{5/2}}{8}(7 + 92x^2 + 96x^4)\, e^{5x^2}$$

$$x^{(6)}(y) = \frac{\pi^3}{8}(127x + 652x^3 + 480x^5)\, e^{6x^2}$$

$$x^{(7)}(y) = \frac{\pi^{7/2}}{16}(127 + 3480x^2 + 10224x^4 + 5760x^6)\, e^{7x^2}, \text{ etc.}$$

Consequently, one can rewrite Eq. (4) as

$$x = \operatorname{fre}(y) = \frac{\sqrt{\pi}}{2}\left(y + \frac{\pi}{2.3!} y^3 + \frac{7\pi^2}{4.5!} y^5 + \frac{127\pi^3}{8.7!} y^7 + \ldots\right). \quad \text{(A-5)}$$

A close investigation of the above results indicates that a term $[\frac{dy}{dx}]^{-1}$ is repeated in each derivative and that the even derivatives become zero at $y = 0$. This leads to writing Eq. (5), the inverse error function, in a tidier form,

$$\operatorname{fre}(y) = \sum_{n=0}^{\infty} f_n(0)\left(\frac{\sqrt{\pi}}{2} y\right)^{2n+1} / (2n+1)! \quad |y| < 1 \quad \text{(A-6)}$$

where, $f_n(0)$ is obtained from the reciprocally defined function, $f_n(t)$, expressed as

$$f_o(t) = 1, \text{ and}$$

$$f_{n+1}(t) = f_n''(t) + 2(4n + 3)\, x\, f_n'(t) + 2[2n+1 + 4(2n+1)(n+1)x^2]\, f_n(t).$$

$$(n = 0, 1, 2, \ldots)$$

Among the analytical properties of the new inverse error function are

$fre(0) = 0$

$fre(-y) = -fre(y)$, and

$fre(1) = \infty$.

In addition, the convergence of Eq. (6) is demonstrated in Figure A-1.

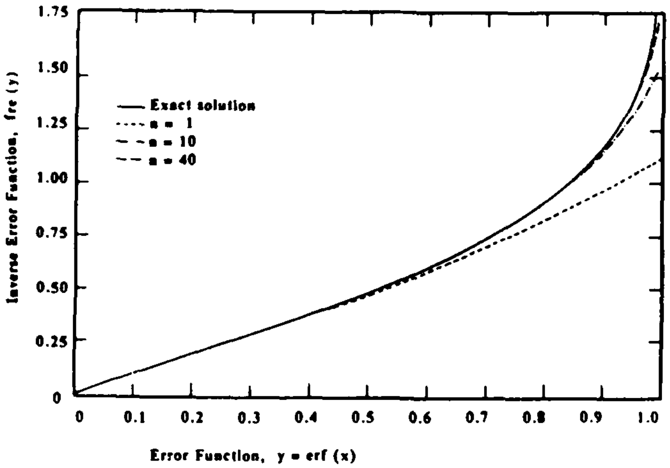

Fig. A-1 Inverse Error Function and its Convergence.

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